

CS2230 Computer Science II: Data Structures

Homework 4

Asymptotic Analysis

Due February 24, 2017, 11:59pm

Goals for this assignment

- Practice using Big-Oh notation
- Analyze the running times of some algorithms

Submission Checklist

You should submit a PDF file titled *hw4.pdf*. Upload it on ICON under Assignments > Homework 4. Physical paper copies are not acceptable.

Part 0: Read about Asymptotic Analysis and take Quiz 4

To help you better understand Big-Oh notation and running time of algorithms, it is recommended that you read Chapter 4 in your textbook.

Part 1: Experiments

1. Ryan and Brandon are arguing about the solution to your upcoming homework assignment on sorting algorithms. Ryan claims that his $O(n \log n)$ -time solution is *always* faster than Brandon's $O(n^2)$ solution. However, Brandon claims that he ran several experiments on both algorithms on his laptop and *sometimes* his was faster. Explain what probably happened.

Part 2: Growth rate

2. Order the following functions by asymptotic growth rate:
 - a. $5n \log n + 4n$ | $12n^2$ | 150 | $4 \log n$
 - b. $12n^4 + 5n$ | 2^{10} | $6 \log n$ | $5n^3$
 - c. 6^n | $7n \log n$ | $8n + 9$ | $60000 \cdot n^6$
 - d. 63 | $64n$ | $3 \log n$ | 2^{n+2} | $10^{\log n}$

Part 3: Proof and Analysis

3. Give a *good* big-Oh characterization in terms of n of the running time of the following. Provide brief justification for your answer (in terms of finding a k and n_0).
 - a. $4n^5 + 3n^3 + 7$
 - b. $15n^{12} + 3n \log n + 2n$
 - c. $3n \log n + 2 \log n + n$
 - d. $12n \cdot 3^n + 50n$
4. Give a *good* big-**Omega** characterization in terms of n of the running time of the following. Provide brief justification for your answer (in terms of finding a k and n_0).
 - a. $5 \log n + 12n^2$
 - b. $6n \log n + 5 \log n + 4n$
5. Show that the following statements are true:
 - a. 4^{n+5} is in $O(4^n)$
 - b. $n \log n$ is in $\Omega(n)$

Part 4: Algorithm Analysis

6. Given the following algorithms below, give a big-Oh characterization of the *running time* in terms of the size of the input, n . Provide justification (description, equations, and/or diagrams) for your answer.

a.

```
public static boolean two_sum(int[] arr) {
    for (int i=0; i<arr.length; i++) {
        for (int j=i; j<arr.length; j++) {
            if (i!=j && arr[i]+arr[j]==0) {
                return true;
            }
        }
    }
    return false;
}
```

b.

```
public static int something(int n){
    for (int i=0; i<42; i++) {
        n += i;
    }
    return n;
}
```

c. First, find the big-Oh running time of inside, in terms of input sizes n_a and n_b .

```
private static double[] inside(double[] a, double[] b) {
    double[] c = new double[a.length];
    int i = 0, j = 0;
    for (int k = 0; k < c.length; k++) {
        if (i < a.length) {
            if (j < b.length) {
                if (a[i] <= b[j]) {
                    c[k] = a[i];
                } else {
                    c[k] = b[j];
                }
            } else {
                c[k] = a[i];
                i++;
            }
        } else {
            if (j < b.length) {
                c[k] = b[j];
                j++;
            }
        }
    }
    return c;
}
```

Now, find the running time of outside, in terms of the size n , using your answer from above.

```
public static double[] outside(double[] list) {
    int x = list.length;
    if (x <= 1) return list;
    double[] a = new double[x/2];
    double[] b = new double[x - x/2];
    for (int i = 0; i < a.length; i++) {
        a[i] = list[i];
    }
    for (int i = 0; i < b.length; i++) {
        b[i] = list[i + x/2];
    }
    return outside(inside(a, b));
}
```

d.

```
int strange_sum(int[] arr) {
    if (arr.length == 1) {
        return arr[0];
    } else {
        int[] arrLeft = new int[arr.length/2+1];
        int[] arrRight = new int[arr.length/2];
        for (int i=0; i<arr.length/2+1; i++) {
            arrLeft[i] = arr[i];
        }
        for (int i=arr.length/2+1; i<arr.length; i++) {
            arrRight[i-(arr.length/2+1)] = arr[i];
        }
        return strange_sum(arrLeft) + strange_sum(arrRight);
    }
}
```

e.

```
public static void printSomething(int n) {
    for(int i = 0; i < n; i++)
    {
        for(int j = n; j > 0; j/=2)
        {
            System.out.println("Something");
        }
    }
}
```